§2. CLASSICAL FIELD THEORY

Newtonian mechanics and the theory of gravity dominated much of the physical sciences into the middle of the 19th century. When applied to astronomy, the mathematics of Newton's equations of motion yielded dramatic results. Problems such as the rise and fall of tides, the flow of rivers, the orbits of projectiles and the motion of machinery were well understood. Compared to the ease of these mechanical problems, the discovery of light and the study of the structure of matter were significantly more difficult. Much of the work on electromagnetism and physical chemistry took place by trial and error. Unlike Newtonian mechanics, these branches of science had no mathematical basis from which to develop. [1]

In the physical sciences, the word field describes a continuous distribution of some type of condition, which pervades a continuum [d'Abr39]. The nature and magnitude of this condition can take on many forms. If the condition can be described by a single valued function for each point in space, then a scalar field is said to exist. A temperature distribution in a volume of gas is an example of a scalar field. In many cases, the condition at each point in space has a direction as well as a magnitude. In this case, a vector field is said to exist. A field of velocities of a fluid in motion is an example of a vector field. The distribution of stress in an elastic medium can be described by a tensor field, as can the gravitational field using a 4–dimensional space–time coordinate system [Sope75].

Before the development of modern electrodynamics, all field theories

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[1] The description of nature using the language of mathematics was introduced by Pythogorus. The Greeks before him envisioned the world in terms of matter. Pythogorus envisioned the world through a mathematical description of form. This form can be described by the fact that matter exists under definite structured conditions and develops or moves according to definite laws, which can be described mathematically.

Mathematics is the way of describing the relationships of these movements. The discovering (by the Greeks) that nature moves according to the laws of mathematics is profound insight into the basic order of the universe. Although Kant has stated that mathematics is merely a *…category of our thinking*, every phenomenon of Nature can be placed in some form of mathematical language [Kant87], [Kant88]. Mathematics is the vocabulary by which the human mind preserves the intrinsic order of nature. All of this classical order will be challenged in later chapters by the mathematics of quantum theory. For now however, chaos does not rule the realm of electrodynamics and classical field theory.

represented conditions of space, usually mechanical categories such as force, velocity or stress. The aether of the electromagnetic field was first described using the notation of a mechanical field theory. After the withdrawal of the aether, the field theory describing electromagnetics still referred to the aether as merely another name for empty space. It was a field of space, which became an active agent instead of a passive void. [2]

§2.1. ELECTRODYNAMICS

The force controlling the interaction of particles, whether charged or neutral, macro or micro in size can be described with the concept of a field of force. Instead of saying that one particle acts on another through direct contact or through some physical medium, as was believed in Newtonian physics, it can be said that the particle creates a field and a certain force resulting from the field, unique to the particle class, then acts on every other particle of the same or similar class, located in this field. In classical physics, the concept of a field is merely a mathematical description of the physical phenomenon—the interaction of particles. [3]

> Here and elsewhere, we shall not obtain the best insights into things until we actually see them growing from the beginning.
>
> — Aristotle

§2.2. ELECTROSTATICS AND EARLY EXPERIMENTS

When a piece of amber is rubbed, it attracts small pieces of material. This discovery is attributed to Thales of Miletus (640–548 B. C.) [Jean25]. [4] A second discovery by Titus Lucretius Carus (~ 99/94–55/51

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[2] The development of field theories took place in the early 19th century only after the development of the theory of partial differential equations [d'Abr39]. Even after the mathematics was available, field theories were not immediately constructed. The mechanical models of stress in elastic media, with their mechanical magnitudes were the basis of much of the work. Only when Maxwell formulated the non–mechanical description of the electromagnetic field did field theories take hold. Maxwell's formulation though depended on the experimental and theoretical results of the pioneers of electrical research, who awaited Maxwell to make the necessary connections before the theory can be turned into practice.

[3] The use of the term particle does not imply subatomic particles, but rather a body of matter that is physically very small, so as to approach a mathematical point.

[4] Thales was considered one of the Seven Sages of Ancient Greece. He was the father of Greek, and consequently of European philosophy and science. His speculations embraced a wide range of topics relating to political and celestial matters.

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B.C.) and described in *De Rerum Natura* (*On the Nature of the Universe*) [Lucr52] was that a mineral ore — lodestone — possessed the ability to attract iron.[5] Plato (428–348/7 B.C.) refers to the attributes of amber in his dialog *Timaues* [Bury29]. By the Middle Ages, the properties of a compressed form of coal called jet had been described by Venerable Bede (673–735).[6] In the 13th century, Pierre de Maricourt demonstrated the existence of two poles in a magnet by tracing the direction of a needle, which was laid onto a neutral magnetized material. His publication described the first observation connected with one of the modern laws of electromagnetics — Gauss's Law for the absence of magnetic charges.

The first use of the word electricity has been attributed to William Gilbert (1544–1603) in 1600.[7] He used electricity to characterize a

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Thales studied astronomy in Egypt which allow him to construct accurate tables forecasting the flooding of the River Nile. He first became widely known by anticipating an eclipse of the sun in May of 585 B.C., which coincided with the final battle in the war between the Lydians and the Persians. He used tables drawn by the Babylonian astronomers, but did not succeed in forecasting the exact day (May 28th) or the hour of the event.

He believed that certain substances, like lodestone (magnetic rock) and the resin amber, possessed psyche (a soul). Many centuries lapsed before Thales' *soul* was identified as static electricity and magnetism. William Gilbert (1544–1603) who had read about the unexplained observation of Thales, also became interested in the intangible property, and decided to call it electricity, from the classical Greek word for amber, which is electron.

[5] Little is known of Lucretius since he lived by the motto of the Epicureans *Live in Obscurity* [Lucr65]. The only contemporary he mentions in his writings was a work of Memmius, who was a politician and praetor in the 58 B.C. *De Rerum Natura*, is a didactic poem on the subject of Nature, creation, and the universe. The basic concepts developed in the poem are that nothing can be created from nothing, nothing can be reduced, the elemental form of matter is tiny particles which are invisible and indivisible. These particles were called *seeds*, *first bodies*, and *first–beginnings*. In modern physics of course, these *particles* are called *atoms*. Lucretius also conjectured that there is also empty space or void. All things in the universe consist of a mixture of particles and this void.

[6] Bede was an English monk who also studied the tides, calculated the dates of Easter for centuries to come and wrote one of the world's great works of history, *The Ecclesiastical History of English*. In this book Bede describes the material jet as..

> ... like amber, when it is warmed by friction, it clings to whatever is applied to it [Bede55].

The cause of this attraction is not well founded, since Bede confuses friction and the warmth produced by friction.

[7] Gilbert was a London physician and president of the Royal College of Surgeons and court physician to Elizabeth I and James I. Gilbert studied the effects of lodestone (magnetite) and introduced the term electric, in his 1600 Latin text *De magnate*,

quantity that many substances share with amber when they are rubbed, including glass, sulfur, wax and certain gems. These observations of electrical attraction led to the idea that electricity was not an intrinsic property of the material, but rather a substance unto itself, which was produced or transferred when the material was rubbed.

Stephen Gray (1670–1736) showed in 1729 that static electricity can be moved between bodies by some substances, among them metals, which are now called conductors [Gray31]. Early laboratory experiments by Benjamin Franklin (1706–90) [Fran51] and Charles–François de Cisternay Du Fay (1698–1739) [Dfay33] established that electrically charged bodies both attract and repel each other, resulting from the presence of negative or positive charges. [8]

Du Fay concluded that there are...

> ...two electricities, very different from each other; one of these I call vitreous electricity; the other resinous electricity [Dfay33].

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magneticisque corporibus, et de magno telluse; physiologia nova, plurimis & argumentis, & experimentis demonstrata (*On the Magnet: Magnetic Bodies also, and On the Great Magnet the Earth; A New Physiology Demonstrated by Many Arguments and Experiments*) after the Greek word elektron (ηλεκτρου) for amber. This book was an attempt to explain the nature of the lodestone and to account for the five movements connected with magnetic phenomena [Gill77]. Gilbert's work is one of the oldest publications on the theory of magnetism. An English translation of Gilbert's work *On the Magnet*, [Pric58] is a facsimile edition of a previous translation [Thom00], which itself is a replica of Gilbert's original Latin edition published in London in 1600.

In his book, Gilbert stated that many substances besides amber could be electrified when rubbed and they would attract light objects. Although Gilbert denied the existence of electrical repulsion, several other researchers observed repulsion later in the 17th century. Niccolò Cabeo (1596–1650) has often been credited with first observing electrical repulsion, but he regarded it as a mere mechanical rebounding of the attracted objects from amber [Home92b]. Gilbert made the distinction between the behavior of dissimilar electrified objects and the behavior of the properties of the lodestone [Heil79]. This separation of electricity from magnetism would be rejoined two centuries later by Øersted in 1813.

Gilbert went on to speculate that magnetism was responsible for holding the planets in their place around the Sun. His improper explanation of orbital mechanics did lay the groundwork for the concept of action–at–a–distance which paved the way for the future concept of universal gravity in the 1680's [Benn80].

[8] It has been known since early times that a piece of amber, when rubbed with fur, will acquire the power to attract ... *feathers, straws, sticks and other small things* [Bari68]. The amber acquires a negative charge when rubbed by the fur, while glass rubbed with silk acquires a positive charge. The exact mechanism involved in transferring a charge (positive or negative) from the surface of one material to another is still not well understood [Wein90], [Moor73].

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The vitreous electricity, from the Latin vitreus for glassy, is produced when glass or crystal is rubbed with silk. The resinous electricity is produced in amber or copal, when they are rubbed with fur. Both of these types of electricity were observed to attract ordinary matter. Vitreous electricity was assumed at attract resinous electricity, but materials containing vitreous electricity were assumed to repel each other and likewise for materials containing resinous electricity.

The strength of this attraction or repulsion is given by an inverse square law. The natural philosophers of the 18th century were disposed to the idea of an inverse square law for electrostatics, following the success of Newton's inverse square law for gravity. It was Charles Augustin de Coulomb's (1736–1806) careful experiments in 1785, using a very sensitive torsion balance, which gave direct quantitative verification of the inverse square law, know as Coulomb's Law [Heav50], [Elli66], [Whit52], [Shan59]. Coulomb stated that

> ... the repulsive forces between two small spheres charged with the same kind of electricity is in inverse ratio to the square of the distance between the center of the two spheres... (and) the law of inverse square was found to hold also the case of attraction.

The inverse square law was found to hold accurately for various charges and separations.[9] This law states that the force between two point charges, q_1 and q_2 — whose dimensions are small compared to their separation — exerted on one another has the direction of the line joining the charges and is inversely proportional to the square of their separation

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[9] The inverse force law for electrostatic charges was first found by Joseph Priestly (1733–1804), who also was the discoverer of oxygen in 1767 [Segr84]. He formulated the inverse square law in a unique manner, which was more convincing that any direct measurement that had been performed at the time. By the same reasoning, Cavendish derived a similar law [Crow62]. After Coulomb's Law had been established, the science of electrostatics became mathematical rather than subjective. The most important problem faced by the scientist of the time was,

> Given the total quantity of electricity on conducting bodies, calculate the distribution of charges on them under the action of their mutual influence and also the forces due to these charges [Born24].

The solution to this problem, usually called the theory of potentials, does not represent the true theory of contiguous action since the differential equations (or difference equations used by Coulomb) describe the charge density as a function of position. They do not describe the transmission of the electrostatic force as a function time. Therefore, they still represent an instantaneous action at a distance.

distance $\mathbf{r}$, [10] as given by,

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[10] The idea that electric forces act like gravity at a distance was first conceived by Æpinus in 1795. He did not succeed in formulating the correct law for the dependence of the electric action at a distance, but he was able to explain the phenomenon of electrostatic induction. The original concept of the inverse square law was discovered through experiment by Henry Cavendish (1731–1810) in 1772. Although Coulomb's method was a direct measurement of the electrostatic force as a function of distance, Cavendish measured the force exerted by a large metallic sphere on a test particle placed inside and outside the sphere. Cavendish made use of the analogy between his experimental apparatus and Newton's description of gravitational forces.

Newton stated that...

> ... a test particle placed anywhere within the sphere will experience no net force exactly when the two–body central force between point charges is exactly proportional to $1/r^2$, while a net force toward or away from the center of the large sphere would be observed if $F(\mathbf{r}) \approx 1/r^{2+e}$ with $\varepsilon \neq 0$.

In Cavendish's laboratory notes, edited by Maxwell and published in 1879, Maxwell concluded ...

> ... that the electric attraction and repulsion is inversely as the square of the distances, or to speak more properly, that the theory will not agree with experiment on the supposition that it varies according to any other law.

Cavendish put an upper bound on ε of $|\varepsilon| \leq 1/60$. Experiments of higher precision involving different sized objects have been performed over the years. Recent results have given error limits on the inverse square relationship of $\varepsilon = (2.7 \pm 3.1) \times 10^{-16}$ for $1/r^{2+\varepsilon}$ [Will71]. The unit of electric charge is the coulomb (C) defined as the electric charge that passes a given point in a wire in one second that is carried by one Ampère of current. The constant f in Eq. (2.1) has the measured value $8.987 \times 10^{9}\, Nm^2 C^{-2}$ [Roll54].

The quantitative work of Coulomb and Cavendish on electrostatics as well as Joseph Black (1728–1799) and Antoine Laurent Lavoiser (1743–1859) and Pierre Simon de Laplace (1749–1827) on heat conduction, and Johann Tobias Meyer (1723–1762), Johann Heinrich Lambert (1728–1777) on magnetism established the process of precise experimentation and the use of mathematical models as the scientific methodology of the nineteenth century [Harm82].

Lavoiser can be considered the Isaac Newton of chemistry (or Newton could be considered the Lavoiser of physics). In his 1787 work *Methode de Nomenclature Chimque,* he restructured chemical theory by organizing and renaming the chemicals of the time. Prior to Lavoiser’s work chemicals were named with colorful but cryptic terms. Ethiop was changed to iron oxide, orpiment to arsenic sulfide. Using prefixes like *ox* and *sulf* and suffixes like *ide* and *ous* he cataloged compounds. Lavoiser’s major contribution was his theory of gases and combustion. He determined that combustion was a chemical process and gases could be compressed which lead to the realization that an element could exist in three states: solid, liquid and vapor.

$$\mathbf{F} = f\frac{q_1 q_2}{\mathbf{r}^2}, \tag{2.1}$$

where the constant f, analogous to Newton's gravitational constant G, has a value which depends on the system of units used to measure the electric charge. [11]

If more charges are added to the expression, the description of $\mathbf{F}$ becomes more complex. Experimentally is can be shown that these new charges have no effect on the interaction of the original two charges.

The effect felt by any charge from other charges in the vicinity can be described by restating Coulomb's Law as,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^3}\mathbf{r}, \tag{2.2}$$

where ε_0 is the permitivity or capacitivity of free space surrounding the charge and $\mathbf{E}$ is the electric field or electric intensity surrounding the charge q_1 — independent of any charges. [12] When another charge q_2 is

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[11] The factor f is independent not only of the condition of the bodies but of their position in space as well. The quantity f has the dimensions of $\text{force}\times(\text{length})^2 \big/ (\text{charge})^2$. Although the force $\mathbf{F}$ is what is measured in the expression, it is useful to introduce the concept of an electric field due to some array of charges. At this point, the electric field can be defined as the force per unit charge acting from a given point in space. This force is a vector function of position, denoted by $\mathbf{E}$. It is assumed that the charge is located at a point. The discreteness of the electric charge means that this mathematical limit is impossible to realize physically. In order to escape from this dilemma, the Dirac delta function can be employed. In one dimension the delta function $\delta(x-a)$ is a mathematically improper function having the properties $\delta(x-a) = 0$ for $x \neq a$ and $\int d\left(x-a\right)dx = 1$ if the region of integration includes a and zero otherwise, and $\int f(x) d(x-a) dx = f(a)$. A rigorous account of the seemingly trivial function is given in [Brac65], [Titc48], [Halp52].

[12] Faraday made a fundamental discovery that the capacitance of a parallel plate capacitor changes when different substances are placed between the plates. A measure of the capacitance is given by the permitivity. If air is the material between the plates, the normalized value of the permitivity is 1.0005. For a vacuum, this value is 1.0. For all substances other than a vacuum, the permitivity between the parallel plates increases in value. Through a series of experiments it was determined that the ratio between the vacuum ε_0 permitivity and the permitivity of various materials ε could be considered a constant, $\varepsilon/\varepsilon_0$, where $\varepsilon_0 = 8.85\times 10^{-12}$ Farad/meter (in MKS units).

The material placed between the plates of the capacitor is called a dielectric — which is usually a good insulator. The increase in the normalized permitivity and the resulting increase in the capacitance is the result of the electrical behavior of the dielectric. The molecules of some dielectrics are non–polar since the average electrical center of their introduced, the electric force it experiences which is generated by the first charge q_1 is given by $\mathbf{F} = q_2\mathbf{E}$.

§2.3. ELECTROMAGNETIC INTERACTIONS

The description of the interaction of electrical matter was first codified by William Watson (1715–1787) and Benjamin Franklin [Fran51], [Cohe41], [Home92]. Franklin stated that there was only one kind of electricity, consisting of extremely subtle particles, which were identified with Du Fay’s vitreous electricity. Franklin proposed that electrical matter differed from ordinary matter in that electrical matter is strongly attracted to ordinary matter and repelled by other electrical matter. Ordinary matter was like a sponge capable of absorbing electrical fluid. When ordinary matter was *squeezed* the electrical matter was *forced out* and became negatively charged. When excess electrical matter was absorbed by ordinary matter it became positively charged [Home92]. Franklin also introduce the concept of conservation of charge, in that electricity is never created or destroyed, only transferred [Fran51]

Franklin's theory of one type of electricity, rather than Du Fay’s two types, was able to describe the mutual repulsion of charged particles as well as the attraction of materials to a charged body. This description accounted for the observed repulsion between materials carrying the vitreous electricity and for the attraction between materials carrying the resinous electricity and the vitreous electricity. What his theory could not describe was the repulsion of two resinous charged bodies. The problem with Franklin’s single type of electricity was addressed by Franz Ulrich Theodor Æpinus (1724–1802) in 1759, followed by Coulomb's explanation

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negative and positive charge is coincident. The molecules of other dielectrics are polar since each molecule acts as a small electric dipole, even in the absence of an external electric field. For non–polar dielectrics the average center of the electrons in the dielectric molecule shifts relative to the positive nucleus containing the molecules protons. For polar dielectrics, the individual molecules are aligned with the external electric field. In both cases, a positive charge appears on the surface of the dielectric nearest the negative plate of the capacitor, while a negative charge appears on the surface of the dielectric nearest the positive plate of the capacitor. If the dielectric material is homogeneous, its interior remains neutral and the material becomes polarized. The effect of the dielectric material is to increase the amount of charge stored in the capacitor. The measured effect is an increase in the normalized permittivity of the capacitor [Roja71]. At a macro level the dielectric constant $\varepsilon/\varepsilon_0$ is a measured constant and is used in the empirical description of Coulomb's Law.

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in 1788.[13]

Æpinus proposed that...

> ... in the absence of a counterbalancing quantity of electricity, ordinary matter repels itself [Aepi59].

The repulsion between materials that carried resinous electricity was explained as the repulsion between materials that had been stripped of their normal amount of electricity. This explanation allowed Franklin’s theory of one type of electricity to account for all the observations made by Du Fay and others in the 18th century. It was not until the discovery of the electron that the full explanation of the forces experienced by electrically charged materials would be available.[14]

§2.4. UNIFYING ELECTRICITY AND MAGNETISM

One problem encountered in the description of electromagnetic force is how to unify differences between electricity and magnetism. Experiment shows that the interaction between bar magnets can be interpreted in terms of the interaction between the poles of the magnets. Early explanations of magnetism were based in two magnetic fluids, the boreal or north and the austral the south. Magnetic particles of the opposite kind attract each other and magnetic fluids of similar kind repeal each other. There were attempts by Coulomb to measure the circulation of the north and south magnetic fluids, but as was common in these early attempts to

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[13] Æpinus described his electrical theory in *Tentamem theoriae electricitatis et magnetisimi* published in St. Petersburg in 1759 [Wein90]. His theory of electricity differs from Franklin's theory published ten years earlier in two ways. Franklin proposed that electrified bodies were surrounded by an electrical atmosphere where Æpinus' theory did not require such a medium. Franklin had also proposed there were two kinds of interparticulate force — a mutual repulsion force between particles of the electrical fluid and an attractive force between particles of the fluid and ordinary matter. Æpinus additionally proposed that particles of ordinary mater mutually repelled each other [Home92a].

[14] Under normal circumstances, electricity is carried by electrons, which Franklin proposed as the electricity of only one type. However, in Franklin’s theory these particles of extreme subtileness carried a positive charge. In fact, the particles carry a charge of the type proposed by Du Fay as resinous, not vitreous electricity. Modern physics follows Franklin by calling vitreous electricity positive and resinous electricity negative, which forever creates the situation where the common carriers of electricity carry a negative electrical charge.

explain nature, they failed. [15]

Each pole, north and south exerts, an inverse square force on the other pole similar to Coulomb's force law between electric charges. It is useful at this point, to picture the poles of a magnet as accumulations of magnetic

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[15] The explanation of magnetism is complicated by the existence of three types of magnetic phenomena — ferromagnetism, paramagentism and diamagentism. The common magnetic attraction of ferrous metals, such as iron filings, to permanent magnets is sufficiently complex to require a quantum mechanical explanation. Substances other than iron, nickel, cobalt and some exotic metals below a temperature of 16° C exhibit magnetic effects. These effects however are many orders of magnitude less than those found in ferromagnetic materials. These small effects can be attractive or repulsive. If a material is placed in a strong external magnetic field and is repelled from the south pole of the field, then it is said to be diamagnetic. If the material is attracted to the south pole of the external field is it said to be paramagnetic.

The qualitative explanation of diamagentism and paramagentism is straightforward. In diamagnetic materials, the atoms have no permanent net angular magnetic moment — in which the electron spins and their orbital motions balance so that any one atom has an average magnetic moment of zero. When an external magnetic field is applied to a diamagnetic material, a current is generated by induction inside the each atom. According to Lenz's law, these currents are in the opposite direction to the applied magnetic field.

In a paramagnetic material the individual atomic magnetic moments are aligned with the external field rather than opposing the field, resulting in an enhancement of the applied field. Paramagnetic forces are small when compared to the mechanical forces caused by the motions of the individual atoms due to the thermal agitation of the material. Because of these thermal excitations, many paramagnetic materials increase their effect at lower temperatures. In any material with atomic magnetic moments, there is a diamagnetic as well as a paramagnetic effect with paramagentism dominating diamagentism [Jack75].

In ferromagnetic materials the net effect of the externally induced magnetic moments are orders of magnitude greater than in diamagnetic and paramagnetic materials. The large magnetic moment in ferromagnetic materials come from the magnetic moment of the electrons in the inner shell of the atom.

When examined at a deeper level each of these qualitative explanations are entirely wrong [Feyn64]. It is not possible to explain the magnetic effects of material without using quantum mechanics. A description of the interaction of the spinning electrons which produce the magnetic moments of the classical magnetism, requires that the interaction generate a strong tendency to align their spin in the same direction. This behavior can be described by Pauli's exclusion principal. Werner Heisenberg proposed a theory in 1928 that ferromagnetism was due to the exchange interaction of the electrons of the incomplete inner shells of iron, nickel, cobalt, etc.

What is remarkable is that the experimentalists of the 18th century were able to describe in sufficient detail the macro–level phenomena of magnetism. It was not until a clear understanding of solid state physics came about that the mechanism of magnetism could be explained in a manner consistent with the experimental evidence. Although these magnetic effects can be measured and theories constructed describing the interaction between the magnetic field and the various magnetic materials, the underlying reason why magnetism exists is still a mystery of nature.

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charge. There are references to magnetic polarizations in many texts, since they are in some ways analogous to the polarization charges found in electrically charged materials [Pais69], [Roja71], [Jack75].

Isolated electrical charges can be studied in the laboratory, while isolated magnetic charges or magnetic monopoles have not yet been observed.[16] In the development of Maxwell's equations, it is useful to

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[16] The quantization of the electric charge is one of the most striking features of atomic physics and there is no explanation for its existence, other than one proposed by P. A. M Dirac. In 1931 Dirac described a theory in which magnetic charges were the consequence of quantizing the electric charge [Dira31], [Krag90], [Amal68], Barr91], [Cabr83], [Carr82a], [Crai82], [Davi88], [Dira48], [Good87], [Lind94], [Yang77]. Isolated magnetic charges or magnetic monopoles were predicted, although there is currently no experimental evidence for them. The discovery of magnetic monopoles would be of great importance and allow the generalization of Maxwell's equations [Efin69], [Lapi69]. Dirac's argument was that the existence of magnetic monopoles could explain the discrete nature of the electric charge.

Theories of matter, which allow magnetic monopoles, are called dual. In the present theory of electromagnetism if the electric and magnetic quantities are exchanged in Maxwell’s equations, a different theory is produced. However, if magnetic charges are assumed to exist, then the exchange of the electric and magnetic terms in the equations results in the symmetric theory [Mule96]. Such a theory is called dual since the two theories are nearly identical. Duality makes elementary and composite particles interchangeable.

The current theory of the formation of the universe predicts the existence of the magnetic monopole [Lind87], [Cabr83], [Carr82]. The monopole would have a mass 1×10^{16} times greater than the proton. These monopoles would have been formed in the very early stages of the big bang and should be as abundant as protons [Carr82a]. The result would be a universe with a density approaching $1\times10^{-29}\, g/cm^3$ which is nearly 1×10^{15} times denser than actual observed — a troublesome result [Lind94]. The current vehicle for the explanation of charge quantization is the Standard Model of particle physics. When several anomalies have been removed from this theory the reason for charge quantization will most likely become clear [Mars93].

There are theoretical issues with admitting magnetic monopoles to the classical electromagnetic theory. If such a monopole were to exist, the curl equation for the magnetic field would be given as, $\nabla\times\mathbf{B}=\rho'$, where ρ' is the magnetic current density, such that $\mathbf{j}'=v\rho'$. However when Maxwell’s equation are written in their potential form, where $\mathbf{B}=\nabla\times\mathbf{A}$ and $\mathbf{E}+\partial\mathbf{A}/\partial t=-\nabla\phi$, it would not be possible for monopoles to exist since the identity $\nabla\cdot\left(\nabla\times\mathbf{A}\right)=0$ results in the monopole versions of the Maxwell curl equations being inconsistent with the normal Maxwell equations. Although Dirac invented the monopole, it was not his aim to do so. The monopole appeared as a result of his calculations.

> It often happens in scientific research that when one is looking for one thing, one is led to discover something else that one wasn’t expecting. This is what happened to me with the monopole concept. I was not searching for anything like monopoles at the time [Dira78].

maintain the symmetry between electricity and magnetism by representing both phenomenon with poles — positive and negative electric charge and north and south magnetic poles. This representation provides a natural extension to the lines of force description of electric and magnetic fields.

The concept of magnetic charge must be used with caution though. The magnetic fields of a bar magnet can be attributed to the electric currents flowing in the magnetic material, either from spinning electrons or form the motion of electrons in the atom [Feyn64] (**II** 36–2).

§2.4.1. LINES OF FORCE

The breakthrough in the theory of electromagnetics came when Hans Christian Øersted (1777–1851) observed direct evidence for electromagnetism.[17] Through experiments with a galvanic cell, he discovered there were electrical forces beyond the static forces previously observed. In 1813, Øersted proposed that electrical force could be transformed into magnetic forces. Through seven years of laboratory experiments, he reached his goal in 1820. Øersted believed...

> ... that magnetic effects are produced by the same powers as electrical ... also that heat and light are produced by the same powers which might be only two different forms of one primordial power [Dibn62].

During the winter of 1819–20 Øersted prepared a lecture on the principals of electricity and magnetism [Shan59]. His usual demonstration was to pass a current generated by a galvanic cell through a thin platina wire and observe the glow produced by the heating of the wire. During the lecture, Øersted placed the wire over a compass. The motion of the compass

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[17] Øersted was born to a village apothecary in Rudkoebing on the Baltic island of Langeland in Denmark. Øersted became an ardent follower of Emanual Kant's (1724–1804) *Naturphilosophie* which was articulated in *General Natural History and Theory of the Heaven*, published in 1755 and *The Critique of Pure Reason*, published in 1781 and again as a second edition in 1787 [Kant81], [Kant88], [Appl95], [Frie92]. This philosophy describes the universe as

> ... a Divine work of art held together by a few simple forces. The basic principle of Øersted's philosophy was that all phenomena are produced by the same force and the science was not merely the discovery of nature; that is the scientist did not just record empirical facts and sum them up in mathematical formulas. Rather, the human mind imposed patterns upon perceptions; and the patterns were scientific. [Oers80].

needle due to the magnetic field produced by the flowing current, was so small that it was first attributed to the confusion surrounding the demonstration equipment.[18] In July of 1820, Øersted resumed his experiments with a stronger galvanic device. The magnetic effect was still small because of the small wires used — limiting the current flowing through the wires. Øersted repeated the experiment with larger diameter wire and discovered that the compass needle was strongly effected by the current flowing in the wire and concluded …

> ... that the magnetic effect of the electrical current has a circular motion round it [Will66].

Other scientists had attempted the same experiment, but had placed the compass needle at right angles to the wire, with no observed effect. They had expected that the magnetic effect should act in the direction of the current. Using this premise, the needle should have been moved to a position parallel to the conducting wire. What Øersted observed however was that the compass needle moved in a direction perpendicular to the wire.[19]

Although Øersted anticipated some form of magnetic action from the conduction of electricity the resulting circular force was both unanticipated and inexplicable [Will71]. Øersted's discovery came as a surprise to the scientific community of the early 1800's. The concept of the helical force surrounding the currently carrying wire was a *hopelessly confusing idea* [Will66], [Plaa94]. The *Naturphilosophie* of the time demanded that all forces be central in nature — a circular force could not be explained. In 1821, the problem appeared to be solved when Ampère proposed the magnetism was simply *electricity in motion*. He observed

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[18] Although it has been popular to write that Øersted's discovery was an accident of scientific experimentation, there is ample evidence to show this was not the case. The exact description of Øersted's experiments were made by Øersted himself in a article on thermo–electricity in the *Edinburgh Encyclopedia* [Will71]. A complete survey of this myth is given in [Stau53] and [Stau57].

[19] Although Øersted would have observed the compass needle move in a perpendicular direction, the needle could have done so only through a small angle before the earth's magnetic field balanced the effect of the magnetic field induced by the current carrying wire [Will83]. The result was that the needle did not end up pointing at right angles to the wire which has been the popular science description of the experiment. In Øersted's original paper he states…

> If the distance of the joining wire from the magnetic needle does not exceed ¾ of an inch, the declination of the needle makes an angle of about 45°.

through an experiment in which two current carrying wires attracted each other when the current flowed in the same direction and repelled each other when the current flowed in opposite directions. Ampère made a revolutionary conclusion that since magnetism is electricity in motion it should be possible to reproduce all the effects of a permanent magnet by various arrangements of current carrying wires. A wire wound in a helix could be made to behave as if it were a bar magnet, with a north and south pole.

After the publication of Øersted's paper on July 21, 1820, [Øers20] attempts were made to explain the motion of the compass needle. Two theories were put forward, one from André–Marie Ampère (1775–1836) [20] and a second by Johann Josef van Prechtl (1778–1854). Although Amprère's theory won out in the end, Prechtl's theory was given some consideration by Faraday in [Fara65]. At this time the various theories of electromagnetism were based on Coulomb's assertion that the interaction between electricity and magnetism was not possible [Will71]. Coulomb's influence was so strong that Ampère was an ardent believer that...

> ... electric and magnetic phenomena are due to "two different fluids" which act independently of each other. [21]

Ampère was in the audience on September 4, 1820, when Dominique François Jean Arago (1786–1853) reported that Øersted had discovered that electricity created magnetic effects surrounding a current carrying wire. Despite what Coulomb had claimed decades before — that electricity and magnetism were independent — Ampère refined Øersted's experiments by neutralizing the effect of the earth's magnetic field.

By placing magnets in suitable locations, the influence of terrestrial magnetism was canceled and the effect of the current induced magnetic field more noticeable [Will83]. He also observed that the compass needle

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[20] Ampère was the son of a well–to–do merchant, who through a self education process laid the foundations for the science of electromagnetics. Ampère educated himself by reading the family library. He survived the French revolution to become a science teacher, first in Lyons then in Bourg. He later took a post at the Ecole Polytechnique, and in 1808 became the inspector general of the university system in Paris. Beginning in 1824, he taught physics at the College de France and philosophy at the Faculte des Lettres. Although successful in his professional career, Ampère's personal live was a tragedy. His father was executed by guillotine during the Revolution and after his first wife's early death, his second marriage was deemed a catastrophe.

[21] Quoted in Arago's *éloge* of Ampère in Domonique François Jean Arago (1786–1853), *Œuvres Complètes* [Will71], Ref: 16, pp.185.

pointed one direction above the wire and an opposite direction below the wire. This lead Ampère to conclude the magnetic force formed a …

> … circle in space, concentric about the wire.[Will83]

Ampère's approach to the problem followed Coulomb's logic — but the results advanced the state of the art. Starting with Coulomb's theory that …

> ... rejected electric and magnetic interaction because these fluids were essentially dissimilar and like only acted on like fluids...

Ampère restated the experimental evidence from a different point of view [Will71], [Will62]. Ampère postulated that since like (only) acts on like the action of the electric current on the compass needle (a magnetic interaction known at the time),

> ... electricity, then, must be the cause of magnetism and the action of the electric current upon the magnetic needle was ... not an interaction between two dissimilar entities ... but rather the action of electricity upon itself [Will71].

Ampère then performed his now famous experiment of passing a current through two parallel wires an observing the attraction and repulsion of each wire to and from the other. [22] These observations were documented in Ampère's memoirs giving birth to the science of electrodynamics.

The fundamental question posed by Ampère and Faraday was whether the magnetic field could be produced by a moving current or whether it

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[22] The details of this experiment are reported in Ampère's memoirs which were collected together by the Société française de physique, *Mémoires sur l'életrodynamique* published in 2 volumes in Paris during 1887–8 [Will71]. The standard high school textbook description of Ampère's experiment shows two parallel wires with a current flowing in the wires. An attractive or repulsive force is then measured between the two wire. This explanation skips over a few of the details of the process. The direction of the current in each wire — relative to the current's direction in the other wire — produces different effects. Since the magnetic field produced by the following current follows the right rule and is circular, currents flowing in similar directions cause the wires to be attracted. Current flowing in opposite directions therefore cause the wires to be repelled. Ampère's experiment was repeated by Faraday during the writing of a survey paper in 1821 and 1822 titled, "Historical Sketch of Electromagnetism," for the *Annals of Philosophy* **18**, pp. 195–200, 274–290, **19**, pp. 107–121 [Good85], [Will85] and an article "On some new Electro–Magnetical Motions, and on the Theory of Magnetism," in the *Quarterly Journal of Science*, **12**, pp. 74–96. During this period Faraday came to grips [Will71] with the theory of electric and magnetic action and confirmed that the magnetic rotation or circular force of the magnetic field was a verifiable phenomena.

also resulted from the interaction of a moving electric charge and a metallic conductor in which the charge traveled. If the later proved correct than magnetic fields would be produced when electric currents moved through a conductor. This problem was stated by Henry Rowland (1848–1901) [23] in a letter to Helmholtz…

> The question I first wish to take up is that of whether it is the mere motion of something through space which produces the magnetic effect of an electric current, or whether those effects are due to some change in the conducting body which, by affecting some medium around the body, produces the magnetic effect [Mill72], [Mill76], [Wear76].

§2.4.2. BEGINNINGS OF FIELD THEORY

A next problem in the description of the electromagnetic force is how to represent the quantitative properties of the electric and magnetic fields. The electric field has the properties of a vector field — direction and magnitude or intensity. The field can be imagined to be seen as lines of force occupying space.

Although there is much debate the origin of field theory [Ners85], this concept can be traced to Michael Faraday (1791–1867) and James Clerk Maxwell (1831–1879) [Ners85], [Will65], [Agas75], [Berk74]. Maxwell interpreted Faraday's work as a replacement of the concept of...

> ... action–at–a–distance with a continuous action. Faraday, ... saw lines of force transversing all space where the mathematicians saw centers of force attracting at a distance... [Maxw65], [Hess61], [Agas71].

The intensity of the electric and magnetic fields can be defined as the number of lines of force passing through a unit area which is at right angles to the direction of the lines of force.

This visualization of the field intensity and lines of force has limited use in today’s physics, since the lines of force do not represent any real

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[23] Henry Rowland was a graduate of Rensselaer Polytechnic Institute in 1870. In 1873 Rowland submitted a paper title “On Magnetic Permeability and the Maximum of Magnetism of Iron, Steel and Nickel,” *Philosophical Magazine*, **46**, 1873, pp. 140–159, to the *American Journal of Science*, on his studies of magnetic permeability. When this paper was rejected he sent it to James Clerk Maxwell who forwarded it to the *Philosophical Magazine*. Rowland’s primary contribution to American science was through his construction of defraction gratings. These gratings advanced the science of spectroscopy in the late 1800’s [Rein64].

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physical phenomenon in electromagnetic theory — however they can be useful in illustrating the underlying mathematics. Faraday used this concept to communicate his discoveries in a way that could be visualized. Even though the lines of force as were used by Faraday may implied a physical mode of the transmission of action, they were never meant to represent any physical lines flowing in space.

Faraday also understood the limitations of this concept when he wrote ...

> The term "lines of magnetic force" is intended to express simply the direction of the force in any given place, and any physical idea or notion of the manner in which the force may be exerted... [Maxw65], [Hess61], [Agas71], [Pari69]. [24]

§2.4.3. Removal of Action at a Distance

One of the great laws of classical physics is Isaac Newton's (1642–1726) law of gravitation. This law embodies the concept of *action–at–a–distance* in which masses exert gravitational force by virtue of their position in

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[24] Throughout the sections dealing with classical electrodynamics, Michael Faraday's influence can be found. Faraday has been described as the Cinderella of science because of his rise from a childhood as the son of a blacksmith in the slums of London to the pinnacle of scientific achievement in the mid 19th century [Wein90], [Whit37], [Will66].

Several biographies have described Faraday's life and scientific accomplishments with [Will71] being one of the better known, so that the details need not be repeated here. In *Experimental Researches in Electricity* published in three volumes and a partial fourth, Faraday figuratively and in some cases literally revolutionized theoretical and experimental physics in the mid 17th century. The results of his efforts are seen all around us, the electric motor, the electric generator, the origins of modern field theory and the basis for the description of most electromagnetic phenomena of the modern world.

Faraday did not start his life as a scientist. As a young man Faraday worked as a booksellers apprentice. While binding books Faraday became engrossed in reading the books particularly books about science. He wrote later that...

> ... there were two that especially helped me, Encyclopedia Britanica, from which I gained my first notions of electricity and Mrs. Jane Marcet's Conversations on Chemistry which gave me my foundations in that science.

Faraday began to attend public lectures in science, many which were given by Sir Humphrey Davy (1790 – 1868). In 1812 Faraday completed his apprenticeship, but did not continue as journeyman bookbinder. Instead, he wrote Sir Davy seeking employment, enclosing notes he had written based on Davy's lectures. Faraday provided the insight necessary to move the theory of magnetism from one based on imponderable fluids to one based on the beginnings of field theory.

space, with the intervening space playing no active role in the conveyance of the gravitational force. In Newton's theory, the gravitational force was conveyed instantaneously to the remote body. It is the instantaneous transmission of force that is the basis of action–at–a–distance. Although Einstein's theories of relativity were to change this concept by the introduction of a finite propagation velocity, both Newtonian and modern field theories provide a mechanism for forces to be felt at a remote location.

Through laboratory experiments, it is observed that the presence of an electric charge produces an electrostatic field, which extends outward through space surrounding the charge. The presence of this field is inferred from the action it has on other electric charges placed in the field. In this manner, action–at–a–distance between charges becomes action–by–contact, where the action is between the remote particle and the field generated by the source charged particle. The basis of electromagnetic theory is that the field does have properties that are associated with matter. The electromagnetic field can posses energy and momentum. The field becomes a dynamical concept and not merely a mathematical concept described by coordinate and time variables [Will66].

A field can now be defined as the representation of the way in which some distributed quantity, the electric or magnetic field behaves. The introduction of the field concept moves the discussion away from the electric and magnetic charges to the behavior of the space surrounding the charges.

§2.5. SPECIAL RELATIVITY AND ELECTROMAGNETIC FIELDS

When the special theory of relativity was introduced in the early 1900's by Albert Einstein (1879–1955), the action–by–contact between the electromagnetic field and charged particles was further altered by the finite propagation velocity of the fields force [Clar71], [Bers73], [Fren79], [Hoff83], [Holt82], [Eins55].[25] A change in the position of one of the

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[25] The theory of relativity is attributed to Albert Einstein and the birth of modern physics. In fact Newtonian physics also contains a relativity principal. In Newton's principal of relativity the speed of the observer was taken for granted, were in Einstein's theory the motion of the observer was explicitly constructed to be consistent with the experimental evidence that the speed of light appears the same not matter at what speed the observer was traveling.

In Einstein's theory there is no meaning to the statement that two separate events occur simultaneously. In order to correct the problem of instantaneous action–at–a–distance Einstein reformulated the concept of force as a field.

charged particles influences other charged particles only after the lapse of a time interval, $\Delta t \cdot c$, where $c \cong 3 \times 10^{10}$ cm/sec. [26] In classical mechanics, the field is merely a mode of description of the physical phenomenon — the interaction of particles [Eisn61]. In the theory of relativity, because of the finite velocity of propagation of the interactions, the forces acting on a particle at a given time are not determined by the positions of the particles at the time a force is felt by a specific particle. A change in position of one particle influences other particles only after a lapse of time. Because of the propagation delay, resulting from the effects of special relativity, the field itself assumes a reality. Therefore, particle interaction is described as an interaction between one particle's action and its generated field and the subsequent interaction of the field with a second particle [Land71].

The two postulates of Einstein's Special Relativity are: (i) The Principal of Relativity, which states that the local laws of physics have the same form in frames of reference that are related by a uniform relative velocity and (ii) The invariance or frame independence of the speed of light, which states that the propagation speed of information or energy has a finite upper bound [Eisn55], [Doug92]. [27] These principals restrict the form of

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[26] Einstein introduced the Special Theory of Relativity in 1905. This theory overturned the Newtonian interpretation of space–time, in which the knowledge of an event propagates at an infinite velocity. He postulated that the laws of physics should be completely objective — that they should be identical for all observers. They should not depend in any way on how the observer is moving relative to an observed object. This can be stated more precisely as: there exists a triply infinite set of equivalent Euclidean reference frames moving with constant velocities in rectilinear paths relative to one another in which all physical phenomena occur in an identical manner. This means that the laws of physics should appear to be identical in an inertial reference frame (non accelerating reference frame) and that all inertial reference frames are equivalent. Einstein also postulated that the speed of light was a universal constant in all inertial reference frames, representing the maximum speed at which the occurrence of an event could be signaled. It is important to remember that it is the laws of physics that are invariant between reference frames. Physical values such as force, electric and magnetic fields and momentum will differ between reference frames.

The Special Theory of Relativity can be restated as:

> The laws of physics cannot provide a way to distinguish one inertial reference frame from another [Tayl63].

[27] Einstein's original paper [Eisn05] submitted in June of 1905, made use of many of the ad hoc assumptions made by his immediate predecessors. This paper placed on a simple conceptual basis, the form of Special Relativity in two postulates. The paper makes no references to the existing literature in particular the work of Lorentz and Poincaré [Lore04]. A balanced description of the contributions of all these authors to the field of Special Relativity is given by [Holt60], [Gold84] and [Gold67]. A more controversial account is given in [Whit60] who states:

the laws of physics and are supported by a vast amount of experimental evidence [Fren68], [Haug87], [Macd80], [Mari82], [Merm84], [Schw84], [Terl68], [Will81], [Will86], [Will87].

The first postulate, when applied to the mechanics of material particles and electromagnetic fields was not a unique feature of the Theory of Special Relativity. This same concept can be used applied in Galileo — Newtonian mechanics. It is the second postulate that revolutionized physics. By limiting the speed of light, the propagation of information and energy, and their associated forces, Einstein redefined the meaning of simultaneity.

This can be illustrated by the following example. Suppose two charged particles, q_1 and q_2 are separated by a distance *d*. If particle q_1 is rapidly moved then returned to rest, it will emit a pulse of electromagnetic radiation which travels at velocity *c* and will be felt by particle q_2 at a time $t=\left(d/c\right)$. If the system of two particles is observed some time after q_1 has come to rest but before the time $t>\left(d/c\right)$, both particles will be observed at rest and would constitute an isolated system whose kinetic energy is zero. At time, $t=\left(d/c\right)$ the second particle q_2 could be observed to move. The energy and momentum of the isolated system would appear to change although there were no external forces acting on it, violating the conservation of energy and momentum. This situation can be reconciled in field theory by observing that there are other forms of energy at work in the system. The other physical entity at work, which contains energy and momentum, is the electromagnetic field [Eyge72]. The energy and momentum transmitted to particle q_2 at time $t=\left(d/c\right)$ is contained in the electromagnetic field. [28]

> ... Einstein published a paper, which set forth the relativity theory of Poincaré and Lorentz, with some amplifications and which attracted much attention [Whit60], pp. 60.

[28] Throughout this book there has been an attempt to present material that is understandable, with a minimum of mathematical background. By using examples such as this it is hoped that the reader will grasp the concepts of Field Theory. However, in reality many of these examples have serious flaws, whose correction requires mathematics, which will be developed later. This example is such a case.

When the observer makes a measurement of the total kinetic energy of the system, consisting of the two particles, the measurement must be made through some form of energy measuring device. This device will depend on the finite propagation velocity of the field conveying the kinetic energy measurements. If the total kinetic energy of the system could be measured without delay for both particles, then the kinetic energy discrepancy

The basis of electromagnetism as a field theory states that charge and currents produced at each point in a field have reality of their own. The field can contain and propagate energy and momentum , which acts on other charges, embedded in the field. The discussion of the interaction between charged particles is now directed to the interaction between one charged particle with the field it generates, and the subsequent interaction of the generated field with a remote charged particle [Heit54].

These two sets of equations describe the consequences of electromagnetic field theory. Maxwell's equations that describe the fields produced by a given set of charges and the Lorentz force equation that describes how a given field acts on these charges.

§2.6. LIGHT — PARTICLE OR WAVE

The nature of light, its propagation though space, its material composition and the formation and absorption by material objects has long been the source of questioning and mystery. Starting with the Biblical creation myth...

> … on the first day God created light.

To the modern creation theory based on the Big Bang [Wein77], light has always been at the origin of scientific questioning. The Greeks [29] based their theory of light on geometric optics and the representation of perspective, the properties of light to reflect and refract, have been part of our culture.

In the 17th century, an era…

> … of change … so radical that classical optics was destroyed and disappeared for good. Today a book on optics written earlier than the seventeenth century would be incomprehensible to a majority of people. [Ronc70].

It was the 17th century instrument, the telescope, that revolutionized the optical world and the methods used to study it. In 1676, Ole Rømer

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would be observed, as described in the example. Since such a measurement is not possible, the real system kinetic energy can not be determined simultaneously for both particles and communicated to a common point, thus the example becomes invalid. Although this example serves to illustrate the concepts behind the force field and its conveyance of energy and momentum, the description of this concept is beyond the mathematics of this monograph.

[29] Aristotle unsuccessfully attempted to interpret the rainbow as a sign of the covenant [Dale78].

(1644–1710) pointed his telescope at Jupiter's innermost moon, Io. He observed that this satellite shows a variation in its motion around Jupiter [Magi65]. Using this instrument, Rømer deduced that the speed of light was 214,300 kilometers per second — about $2/3$'s of the modern value.

In 1666, at the age of 23, Isaac Newton began his experiments with sunlight passing through a triangular glass prism. Newton showed …

> … Colours are not "Qualifications of Light" derived from refractions or reflections of natural bodies (as 'tis generally believed) but "Original and connale properties" … [white light] is not similar to homogenial but consists of Difforn rays, some of which are more Refrangible than other … [it is] a confused aggregate of rays induced with all sorts of colours as they were promiscuously darted from the various parts of luminous bodies. [Newt72]

Newton recorded his feelings in *Opticks* [Shap84], [Newt31]. One interesting conjecture in *Opticks*, which pertains to the subject mater here, is summarized in Newton's 29th Query:

> Are not the Rays of light very mall bodies emitted from shining substances? … Nothing more is required for producing all the variety of Colours, and degrees of Refragibility, than that the Rays of light be bodies of different Sizes, the least of which may make violet the weakest and darkest of the Colours, and be more easily diverted by refracting Surfaces from the right Course; and the rest as they are bigger and bigger may make the stringer and more lucid Colours, blue, green, yellow and red, and be more and more difficulty diverted.

In 1690 an alternative to Newton's description of light as small bullets was put forth by Christian Huyghen's (1629–1693) [Huyg90]. Huyghen's theory proposed that …

> … Light spreads as sound does, by spherical surface waves. I call them waves from their resemblance to those which are seen to be found in water when a stone is thrown into it. [30]

Not until 1925 did Newton's description of light begin to fail and the corpuscular theory of Newton and the wave theory of Huyghen's merge into the quantum theory of light.

Underlying this revolution in the description of light, the concept of the

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[30] Huyghens postulated that the waves of light traveled through a medium, just as the wave on the surface of the water.

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aether was also undone. The notion of the all–prevasive ether (aether) goes back to Plato.

> Nature's abhorrence of a vacuum was a sufficient reason for imagining an all surrounding aether … aether's were invented for plants to swim in, to constitute electric atmospheres and magnetic effluvia [Niven].

§2.7. OVERVIEW OF THE WAVE EQUATION

The next chapters present Maxwell's equations and their solutions in the form of the electromagnetic wave equation. The mathematical description if waves is at the heart of many fields in physics. The foundations of the theory of waves can be found in almost any good physics text. This section will provide a brief review of the equations of a traveling wave and prepare the reader for the description of Maxwell's wave equation.

The development of the mathematics of the *wave equation* can be traced to the Greek's. A vibrating string moves much too fast for the human eye to see the actual shape of the string. The Greek's however showed that the pitch at which a string vibrates depends on the position of the *nodes* [Stew95]. These nodes are locations along the string where the motion of the string is stationary. At the string's fundamental frequency, only the end points of the string are stationary. If the center of the string is held stationary, when it is *plucked* it will vibrate at one octave higher. If a point $1/3$ the length is held stationary, the string will vibrate in two modes. The waves of the vibrating string are *stationary* waves that do not travel along the string, but only move *up and down*. These waves are transverse waves.

In 1715, Brook Taylor (1635–1731) published a paper *Methodus incrementorum* in which he described the theorem on power series expansions. Taylor also wrote on the theory of the vibrations of a string, in terms of its length, tension and mass. In 1743, Jean le Rond d'Alembert (1717–1783) published *Traité de Dynamique*, in which he developed Newton's dynamics using the concept of energy rather than force. He later described the standing waves in violin strings as composed of *any shaped wave*.

In 1748, Leonhard Euler (1707–1783) described in *Introductio in Analysin Infinitorum* the wave equation of a vibrating string using a differential equation. Euler's solution to the wave equation is based on the superposition of two arbitrary shaped waves, each traveling in opposite directions along the vibrating string. When these two waveforms are combined, Euler claimed that the motion of the string could be described if the ends of the string are fixed.

Daniel Bernoulli (1700–1782) solved the wave equation in a different manner. In Bernoulli's method, the motion of the string could be described as the superposition of an infinite number of sinusoidal waves. The superposition principal was used by Fourier to describe the series expansion of a periodic function in terms of sine's and cosine's.

The general form the wave equation is given by,

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0\,, \tag{2.3}$$

where u is the disturbance that is being propagated and v is the velocity of the propagation. In general $v = f\lambda$, where f is the frequency of the propagated disturbance and λ is the wavelength of the propagated disturbance. Using these definitions, $\mathbf{k} = 2\pi/\lambda$ and $\omega = 2\pi f$. The relationships between the spatial oscillation rate ($\mathbf{k}$) and the temporal oscillation rate (ω) is called the dispersion.

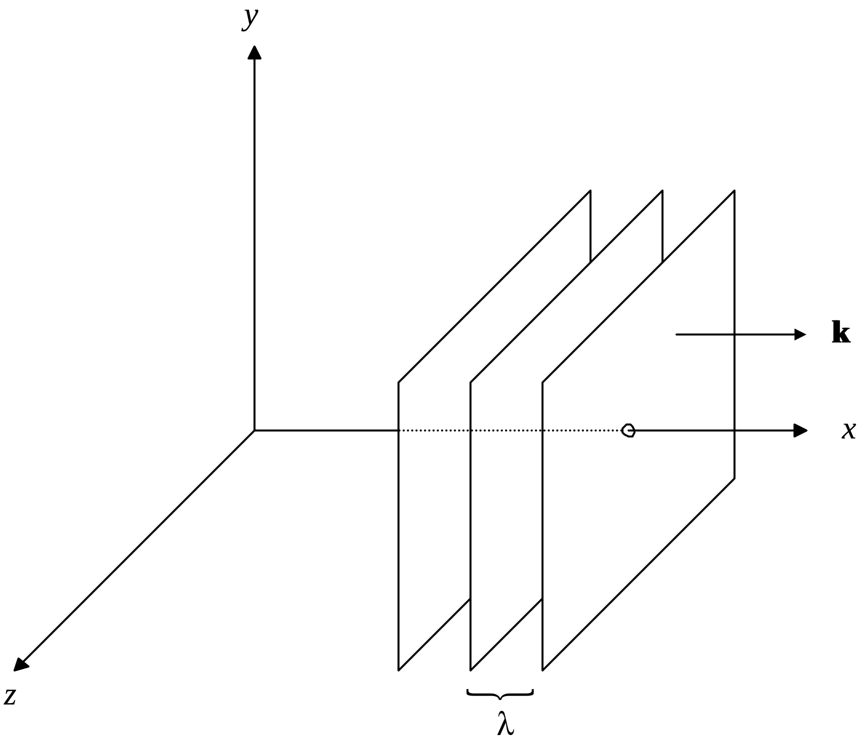

Figure 2.1 — A plane wave front traveling in the x–direction. The wave

fronts spaced λ distance apart with a constant phase $\left(\omega t - kx + \phi\right)$.

By choosing a specific coordinate axis for the propagation direction a *propagation vector*, $\mathbf{k}$, can be used to describe the motion of the wave through space. This propagation vector, $\mathbf{k}$, is the number of cycles of the wave in a unit, whose normalized length is $k = \left\|\mathbf{k}\right\|$. The propagation vector, $\mathbf{k}$, is perpendicular to the wave front.

Since the use of the plane wave description may entail combining waves, which have different coordinate systems, the selection of a single coordinate system may not be possible. Two other vectors are also needed for this arbitrary coordinate system description. The direction of the wave fronts — in a single dimension — will be given by $\mathbf{x}$.

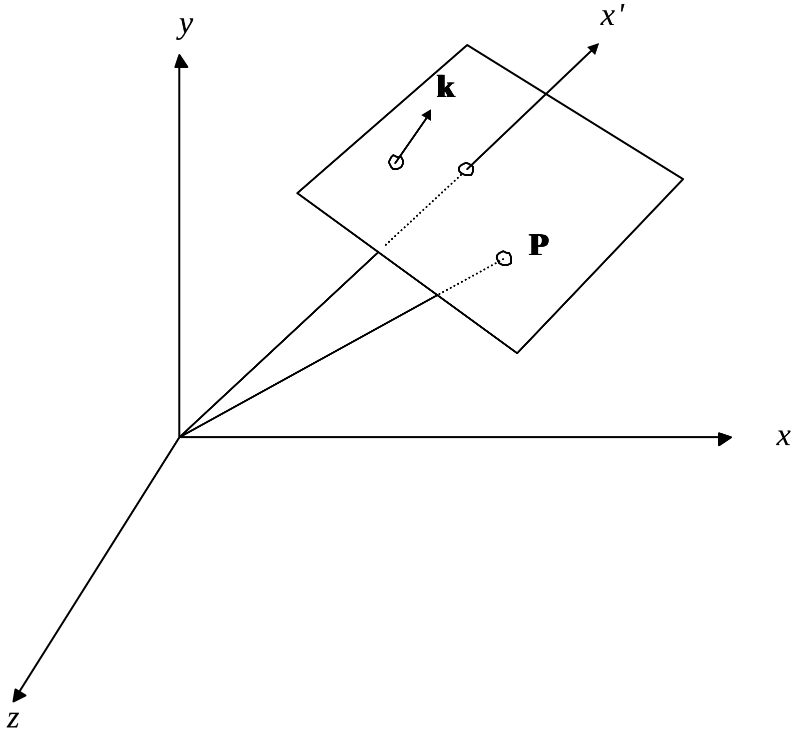

Figure 2.2 — A Section of a plane wave front traveling along the *x*' axis. The coordinates *x*', *y*', and *z*' are an arbitrary angle relative to the original coordinates *x, y* and *z*. The radius vector **r** joins the origin with a point **P** on the wave front. The propagation vector **k** is parallel with the coordinate vector *x*'.

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This direction can in fact be any of the three dimensional coordinates, but **x** is a common one. The second vector that connects the origin of the propagating plane wave with the wave front is **r**, and is usually called the *radius* vector. The propagation vector, **k**, lies along the wave front's direction vector **x**. The radius vector can be defined as $\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$, since it represents a point on a given wave front as measured from the origin. In this definition the vectors **x**, **y**, and **z** are the vectors along each Cartesian coordinate system axis. [31] The wave front is then described by,

$$\left(\omega t - k\mathbf{x}' + \varphi\right) = \text{Constant}, \tag{2.4}$$

where φ is a phase factor.

Note that $\mathbf{x}'$ is a different coordinate that the **x** coordinate of the Cartesian coordinate system centered on the origin. This plane wave can be described in a new coordinate system (unprimed) by the plane wave,

$$\left[\omega t - k\left(\mathbf{r} \cdot \mathbf{x}'\right) + \varphi\right] = \text{Constant} \tag{2.5}$$

Using some vector algebra gives,

$$k\left(\mathbf{r} \cdot \mathbf{x}'\right) = \left(k\mathbf{x}'\right) \cdot \mathbf{r}. \tag{2.6}$$

The product $k\mathbf{x}'$ is a vector lying in the same direction as $\mathbf{x}'$, whose magnitude is $k = \omega/v$. This is the final definition of the propagation vector **k**.

The disturbance traveling in the $\mathbf{x}'$ direction can now be described by,

$$u = u_0 \cos\left(\omega t - \mathbf{k}x + \varphi\right). \tag{2.7}$$

The wave equation given in Eq. (2.3) does not state how many dimensions are in coordinate system of the disturbance. It will be assumed that the wave is propagating in three dimensions. However, in many of the examples given later, the wave equation will be simplified to one dimension. Without any loss in its descriptive power.

In principal, there are two kinds of waves, (i) longitudinal waves in which $k \parallel v \parallel u$ and (ii) transversal waves in which $k \parallel v \perp u$. It will be the transversal waves that will be of interest in the study of the propagating electromagnetic field.

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[31] In most texts this expression is given by $\nabla \cdot (\nabla \times \mathbf{A}) = 0$, where *x*, *y* and *z* are unit vectors in the vector space of the coordinate system.

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The phase velocity of the propagating disturbance is given by,

$$v_{phase} = \frac{\omega}{k}. \tag{2.8}$$

The group velocity is given by,

$$v_{group} = \frac{d\omega}{dk} = v_{phase} + f\,\frac{dv_{phase}}{dk} = v_{phase}\left(1 - \frac{k}{n}\frac{dn}{dk}\right), \tag{2.9}$$

where n is the refractive index of the medium through which the disturbance is propagating. Information transferred by the disturbance by modulating it does so at the group velocity. If v_{phase} does not depend on ω, then $v_{phase} = v_{group}$.

I have also a paper afloat, containing an electromagnetic theory of light, which till I am convinced to the contrary, I hold to be great guns.

— James Clerk Maxwell

in a letter to his cousin Charles Cay [Ever70]