A MATHEMATICAL TOUR OF THE ELECTROMAGNETIC FORCE

from

Michael Faraday to Quantum Field Theory

There are physical phenomena in everyday life that are taken for granted simply because the explanation of their behavior closely matches the expectations of the observer. For some of these phenomenon, an extensive body of theoretical knowledge exists which matches the experimental observations. The electromagnetic force is one of these phenomenon. The observer can envision empty space filled with electromagnetic waves, and describe these waves and their effects on matter with mathematical precision. Devices can be constructed, based on electromagnetic theory, that confirm our *belief* that the electromagnetic phenomena are well understood — that is, observations are produced consistent with expectations. With further investigation new questions arise, requiring a reformulation of the theory which supports these observations.

The classical electromagnetic field is described by Maxwell's equations. From these equations much of the material world can be analyzed. At physical sizes below molecules Maxwell's description of nature becomes unusable. A quantum mechanical description of the electromagnetic field is required.

Such a description is provided by Quantum Electrodynamics. Starting with the classical description of the radiated electromagnetic field, this book makes use of a simple human experience — the receipt of radio signals — to explore the mathematical foundations of the electrodynamics.

Starting with the earliest experiments in electrostatics, Faraday, Maxwell and Hertzian formulations of the radiated field are described. The theory of antennas and electromagnetic reflection and refraction are explored. All of this material is a prelude to the quantum mechanical description of the electromagnetic field and its interaction with matter. In this description, the quantized field interacts with charged particles through the exchange of a particle which carries the electromagnetic force through free space — the photon. The behavior of this interaction at the quantum mechanical level provides new insight to the complexities of nature.

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Why Study Mathematics?

I am amused, I said, at your fear of the world, which makes you guard against the appliance of insisting upon useless studies; and I quite admit the difficulty of believing that in every man there is an eye of the soul which, when by other pursuits lost and dimmed, is by there purified and re–illuminated and is more precious for then ten thousand bodily eye, for by it alone is truth seen.

— Socrates to Glaucon in *Plato's Republic Book VII*

Physics ... is essentially an intuitive and concrete science. Mathematics is only a means for expressing the laws that govern phenomena.

— Einstein to Solovine in [Solo79]

Table of Contents

Table of Contents

I write to discover what I think.
— Daniel J. Boorstin, Librarian of Congress